Difference Equations

24†. We have seen in lectures that if the root of the auxiliary equation has repeated roots then the solution has the form

$$x_n = A\lambda^n + Bn\lambda^n.$$

In this question we will see where the n comes from.

(a) Write the following sum as a single fraction.

$$\frac{A}{x-c} + \frac{B}{(x-c)^2}$$

(b) We know that

$$\frac{1}{1-\lambda} = 1 + \lambda + \lambda^2 + \cdots$$

By differentiating both sides with respect to λ find an expansion for $1/(1-\lambda)^2$.

(Do you see where the n is coming from?)

(c) Suppose that our difference equation is

$$x_{n+2} - 4x_{n+1} + 4x_n = 0,$$

where $x_0 = 5$ and $x_1 = 8$. Using the methods of the last tutorial show that

$$f(t) = \frac{5-12t}{(1-2t)^2}.$$

(d) Using the result of part (a) show that

$$f(t) = \frac{6}{1-2t} - \frac{1}{(1-2t)^2}.$$

(e) By expanding the right hand side and considering the coefficient of t^n show that

$$x_n = 6 \times 2^n - (n+1) \times 2^n.$$

and hence deduce that $x_n = (5-n)2^n$.