Student Name: _____

Student Number: _____

AUSTRALIAN CATHOLIC UNIVERSITY

Castle Hill

Semester II, September 1997

This paper may not be taken from the examination room

MM303: Abstract Algebra

W. N. Franzsen

Time: 1 hour 30 minutes, plus 10 minutes reading time

Instructions:

There are 10 questions on 3 pages. Complete, careful, correct answers to 8 questions will obtain full marks. The marks available for each question are indicated at the end of each question.

This examination is worth 30% of your final mark.

You must give reasons, if none are given then you will get no marks for that part of the question.

You may attempt any and all questions in any order you choose.

The question paper must be handed in with your answers.

1. Prove that if $\mathbb{F}$ is a field, a, $b \in \mathbb{F}$ and $ab = 0$ then $a = 0$ or $b = 0$.

(15)

2. Show that any subfield of $\mathbb{C}$ is an extension of $\mathbb{Q}$.

(10)

3. Show that $\sqrt[3]{2} + \sqrt{3}$ is irrational.

(25)

4. Evaluate $\big[\mathbb{Q}[\sqrt{2} + \sqrt{3}\,] : \mathbb{Q}\big]$.

(25)

5. Carefully, and in complete detail, define each of the following terms or expressions.

(a) Degree of a polynomial.

(b) α is an algebraic number.

(c) β is an irrational number.

(d) Ring.

(e) $\mathbb{K}$ is an extension of $\mathbb{F}$.

(20)

6. Evaluate $\deg\big(\sqrt{2}, \mathbb{Q}\big)$.

(10)

7. Show that

$$\left\{ \begin{pmatrix} 2 \\ 1+i \end{pmatrix}, \begin{pmatrix} 3-i \\ i-2 \end{pmatrix}, \begin{pmatrix} 1+i \\ 1+i \end{pmatrix}, \begin{pmatrix} 1+i \\ 1-i \end{pmatrix} \right\}$$

is a basis for $\mathbb{C}^2$ over $\mathbb{R}$.

(15)

8. (a) Show that if α is algebraic over $\mathbb{F}$ then there is a monic polynomial $f(X) \in \mathbb{F}[X]$ with α as a root.

(b) If α is algebraic over $\mathbb{F}$ must $\alpha \in \mathbb{F}$?

(15)

9. Factorise the following polynomials into irreducibles over the given field.

(a) $2X^5 - X^4 + X^3 + 2X^2 - 8X - 6$ over $\mathbb{Q}$.

(b) $X^4 + X^3 - X^2 + 6$ over $\mathbb{R}$.

(c) $X^3 + X^2 + X + 1$ over $\mathbb{C}$.

(25)

10. If $\mathbb{L}$ is a field show that

(a) $a \times 0 = 0$, $\forall a \in \mathbb{L}$.

(b) $(-a) \times (-b) = ab$, $\forall a, b \in \mathbb{L}$.

(40)