AUSTRALIAN CATHOLIC UNIVERSITY

Castle Hill

Semester IV, 1994

MM203: Linear Algebra

LECTURER: W. N. Franzsen

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TIME ALLOWED: 2 hours, including 10 minutes reading time

INSTRUCTIONS TO STUDENTS

There are 10 questions on 3 pages. Complete, careful, correct answers to 8 questions will obtain full marks. The marks available for each question are indicated at the end of each question.

This examination is worth 40% of your final mark.

You must give reasons, if none are given then you will get no marks for that part of the question.

You may attempt any and all questions.

Calculators are permitted.

The question paper must be handed in with your answers.

1. (a) If $\underset{\sim}{u}$ is a vector and λ is a scalar define $\lambda\underset{\sim}{u}$.

(b) Show that if $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are not parallel then there is no scalar such that $\underset{\sim}{u} = \mu\underset{\sim}{v}$.

(c) Hence, or otherwise, show that $\{\, \underset{\sim}{u}, \underset{\sim}{v} \,\}$ is linearly independent if and only if $\underset{\sim}{u}$ is not parallel to $\underset{\sim}{v}$.

(30)

2. (a) Find λ and μ such that

$$\lambda \begin{pmatrix} 9 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 7 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}.$$

(b) Find the point M which is the centroid of the triangle with vertices $(0, 0)$, $(1, 4)$ and $(-1, 2)$. (The centroid is the point where the medians meet.)

(15)

3. (a) Show that $\underset{\sim}{u} \cdot \underset{\sim}{u} = \|\underset{\sim}{u}\|^2$.

(b) Find three vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\underset{\sim}{w}$ such that

$$\begin{aligned} \underset{\sim}{u} \cdot \underset{\sim}{v} &= 1 \\ \underset{\sim}{v} \cdot \underset{\sim}{w} &= 2 \\ \underset{\sim}{w} \cdot \underset{\sim}{u} &= 3 \end{aligned}$$

or show that it is not possible.

(20)

4. Find the equation of each of the following lines. (The parametric equation will do.)

(a) The line through $(1, 5)$ with direction vector $\begin{pmatrix} 9 \\ 10 \end{pmatrix}$.

(b) The line through $(-1, 3, 7)$ and $(4, 4, 4)$.

(c) The line through $(2, 0, 1)$ which is parallel to the line

$$\frac{x-1}{2} = y + 2 = \frac{z-4}{-3}.$$

Do the last two lines intersect?

(20)

5. Find the equation of the plane through the points $(1, 2, 3)$, $(4, 5, 6)$ and $(7, 9, 8)$. Hence write down a normal to this plane.

(15)

6. We know that

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} u_2v_3 - u_3v_2 \\ -u_1v_3 + u_3v_1 \\ u_1v_2 - u_2v_1 \end{pmatrix}.$$

(a) Show that $\underset{\sim}{u} \times \underset{\sim}{v}$ is perpendicular to $\underset{\sim}{u}$ and $\underset{\sim}{v}$ most of the time. (This requires you to say what 'most of the time' means.)

(b) If $\underset{\sim}{u} \times \underset{\sim}{v} = \underset{\sim}{0}$ what can we say about $\underset{\sim}{u}$ and $\underset{\sim}{v}$?

(20)

7. Consider the following matrices;

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 3 & -2 \\ 2 & 8 & 8 \end{bmatrix} \qquad C = \begin{bmatrix} -2 & 3 \\ 1 & 1 \\ -3 & -3 \\ 0 & 4 \\ 6 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 3 \end{bmatrix}.$$

There are 16 possible ways of multiplying two of these: AA, AB, AC, AD, BA, BB, …. Evaluate all of these products which are defined.

(20)

8. Find the inverses of the following matrices (if possible).

(*i*) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

(*ii*) $\begin{bmatrix} 2 & 3 \\ 10 & 15 \end{bmatrix}$

(*iii*) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

(10)

9. (a) Evaluate $\det \begin{bmatrix} 1 & a \\ 1 & b \end{bmatrix}$

(b) Expand $(b-a)(c-a)(c-b)$.

(c) Evaluate $\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$

(d) Based on the last three parts write down

$$\det \begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}.$$

(10)

10. (a) Show that if A and B are invertible then so is AB.

(b) What is $(AB)^{-1}$ in this case?

(c) What can you say about the matrix A if

$$A^2 = A.$$

(40)