

AUSTRALIAN CATHOLIC UNIVERSITY

Castle Hill

Semester II

MM103: Calculus I

LECTURER: W N Franzsen

September 1993

TIME ALLOWED: 2 hours, including 10 minutes reading time

INSTRUCTIONS TO STUDENTS

*There are 10 questions on 3 pages, you may attempt any and all questions. Complete, careful, correct solutions to 8 questions will obtain full marks. The number of marks available for each is given at the end of the question.*

*Non-programmable calculators are permitted.*

**All** reasons must be given, in none are offered then you will receive no marks for that part of the question.

*You may not leave the room during the first 30 minutes of the examination nor during the last 10 minutes.*

*Hand in the question paper with your attempt.*

1. (a) Define the term ‘the derivative of a function’.

(b) Find the tangent to the curve  $y = x^4 - x^2 + 5$ . (10)

2. (a) Write down the index laws, including any definitions you need to ensure that these laws work for all indices.

(b) Let  $y = x^{\frac{1}{2}}$ , then  $y^2 = x$ . Differentiate implicitly and show that  $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ .

(c) Repeat part (b) for  $y = x^{\frac{7}{5}}$ . (25)

**3.** Find all the stationary points of the following functions and determine their nature.

(a)  $f(x) = x^5 - 5x$

(b)  $f(x) = 2x^5 + 5x^4 - 10x^2 - 10x + 1$

(c)  $f(x) = x^3 + 3x$

(20)

**4.** Sketch the following curves showing all important features.

(a)  $y = 2x^3 + 15x^2 + 36x - 1$

(b)  $y = x^5 - 5x + 1$

(20)

**5.** Differentiate the following functions.

(a)  $f(x) = x(\sin x + \cos x)$

(b)  $f(x) = \sin x \times (x + \cos x)$

(c)  $f(x) = \frac{x}{x+3}$

(d)  $f(x) = \left(\frac{x}{x+3}\right)^{\frac{1}{2}}$

(e)  $f(x) = \sqrt{\sin x + \cos x}$

(f)  $f(x) = x \cos x - x^2 \tan x$

(g)  $f(x) = \operatorname{cosec} x$

(15)

**6.** Consider the line with equation  $y = mx + c$ . Without using differentiation show that the gradient of this line is equal to  $m$  everywhere. (Hint pick two points with  $x$ -coordinates  $a$  and  $b$  respectively.)

(25)

7. Prove the following are true for  $m$  and  $n$  positive integers,  $m > n$ .

$$\frac{a^m}{a^n} = a^{m-n}$$
$$(ab)^m = a^m b^m$$

State explicitly any definitions you need.

(10)

8. We know that

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

Show that

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$
$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

Hence deduce the rules for  $\cos(x \pm y)$ .

(20)

9. Given two numbers which add up to 10, what is the largest possible value for their product?

(15)

10. (a) Using the product rule, the chain rule and the fact that if  $y = \frac{u}{v}$  then  $y = u \times (v)^{-1}$ , derive the quotient rule.

$$\frac{dy}{dx} = \frac{\frac{du}{dx} \times v - u \times \frac{dv}{dx}}{v^2}.$$

(b) Using the quotient rule, the chain rule and the fact that if  $y = uv$  then  $y = \frac{u}{v^{-1}}$ , derive the product rule.

$$\frac{dy}{dx} = \frac{du}{dx} \times v + u \times \frac{dv}{dx}.$$

(c) We know that for most functions  $y = u \times v$

$$\frac{dy}{dx} \neq \frac{du}{dx} \times \frac{dv}{dx}.$$

Can you find two functions  $u$  and  $v$  for which it is true that if  $y = uv$  the

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dv}{dx}?$$

(40)