AUSTRALIAN CATHOLIC UNIVERSITY

Castle Hill

Semester II, 1992

MA103: Calculus I

LECTURER: W. N. Franzsen

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TIME ALLOWED: 2 hours, including 10 minutes reading time

INSTRUCTIONS TO STUDENTS

There are 10 questions on 3 pages plus a sheet of diagrams to be included in your answer booklets if you attempt the appropriate question. Complete, correct answers to 7 questions will obtain full marks. The marks available for each question are indicated at the end of each question.

This examination is worth 30% of your final mark.

You must give reasons, if none are given then you will get no marks for that part of the question.

Calculators are permitted.

You may attempt any and all questions.

The question paper must be handed in with your answers.

No-one may leave the exam room during the first thirty minutes or during the final fifteen minutes.

1. Find the equations of the following straight lines.

(a) The line joining $(2, 7)$ and $(-1, 5)$.

(b) The line with gradient -1 which passes through $(1, 1)$.

(c) The line through $(2, 7)$ which is parallel to the x-axis.

(d) The line through the points A and B, where A is the x-intercept of the line $y = 2x+4$ and B is the point of intersection of the lines in parts (a) and (b).

Note: In part (d) you will not be penalised twice. If you have made mistakes in either part (a) or (b) you can still get full marks in part (d) if your answer is consistent with the answers you got in the first two parts.

(15)

2. (a) Find the equation of the tangent to the curve $y = 3x^3 - 5x^2 + 2x - 1$ at the point $(1, -1)$.

(b) The parabola $y = ax^2 + bx$ passes through the point $(1, 3)$ and has tangent $y = 2x + 1$ at that point. Find a and b.

(20)

3. Factorise the following quadratic expressions.

(a) $x^2 - 3x + 2$ (b) $x^2 - 4x + 4$

(c) $6x^2 - 5x - 6$ (d) $x^2 - 49$

(e) $x^2 + 49$ (f) $\frac{1}{6}x^2 - x - 12$

(10)

4. On the attached sheet you will find sketches of some functions. On the axes below each one draw a rough sketch of the gradient function of each. **Remember to include this sheet in your answer book**. Also put your name on it in case the sheet gets separated from your answer booklet.

(20)

5. Differentiate the following functions.

(a) $f(x) = \sin x$ (b) $f(x) = \cos x$

(c) $f(x) = \tan x$ (d) $f(x) = x^3 - 4x^4 + x^5 - 2$

(e) $f(x) = x^3 + x^2 + x + 1$ (f) $f(x) = \dfrac{x^2 + 2x + 1}{x^3 + 1}$

(g) $f(x) = x \sin x$ (h) $f(x) = \cos\left(2x^2 - x^3\right)$

(i) $f(x) = \dfrac{\sin x}{x^2 + 1}$ (j) $f(x) = \tan\left(x \sin x\right)$

(20)

6. Find all the stationary points of the following functions and determine their nature.

(a) $f(x) = 4x^2 - 4x + 1$

(b) $f(x) = x^3 + 9x^2 - 21x + 2$

(c) $f(x) = \dfrac{2x^2 + 3x}{x + 1}$

(25)

7. (a) Write down the 4 index laws. (Or, if you think there are more than 4, write down the 4 you consider most important.)

(b) Hence or otherwise simplify the following. (Otherwise does not include just writing down the answer, there must be some work shown.)

(i) $\dfrac{x^3 \times x^4}{x^{-3}}$ (ii) $\sqrt[3]{\dfrac{x^{\frac{1}{4}} \times x^{\frac{2}{3}}}{x^{\frac{3}{2}}}}$

(iii) $\left(y^2 \times x^3 \times y^{-\frac{1}{2}}\right)^3$ (iv) $\left(\dfrac{\sqrt[3]{x} \times x^{\frac{2}{3}}}{\sqrt{x^{-2}}}\right)^{\frac{1}{4}}$

(v) $\left(\left(x^2\right)^2\right)^2$

(10)

8. Sketch the following curves indicating all important points.

(a) $y = x^3 - 6x^2 + 9x + 2$

(b) $y = x^3 - 3x^2 + 3x - 1$

(25)

9. Find $\dfrac{dy}{dx}$ in terms of x and y given:

(a) $xy = x^2 + 1$ (b) $x^3 - y^3 = 3$

(c) $\dfrac{x}{y} = \dfrac{y}{x}$ (d) $x^2 + xy + y^2 = 14$

(15)

10. Let $f(x)$ be a function which is differentiable everywhere. We wish to find a root of the equation $y = f(x)$. Suppose $x = a$ is close to such a root.

(a) Find the equation of the tangent to the curve $y = f(x)$ at the point where $x = a$.

(b) Find where this tangent hits the x-axis. Hence deduce Newton's formula for finding a closer approximation to a root of $f(x)$.

(c) Can you find a condition on $f(x)$ for this new approximation to be closer to the root than a?

(40)