Student Name: _____

Student Number: _____

AUSTRALIAN CATHOLIC UNIVERSITY

Castle Hill

Semester II, November 1997

This paper may not be taken from the examination room

MM303: Abstract Algebra

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Time: 2 hours, plus 10 minutes reading time

Instructions:

There are 10 questions on 3 pages. Complete, careful, correct answers to 8 questions will obtain full marks. The marks available for each question are indicated at the end of each question.

This examination is worth 50% of your final mark.

You must give reasons, if none are given then you will get no marks for that part of the question.

You may attempt any and all questions in any order you choose.

The question paper must be handed in with your answers.

1. Show that the number $\dfrac{\sqrt{17-2\sqrt{3}}}{\sqrt[4]{1+\sqrt{11}}}$ is constructible.

What can we say about the degree? (I do *not* want you to calculate the degree, I only want a list of the possible values.)

(20)

2. Find a basis for the extension $\mathbb{Q}\big[\sqrt{2+\sqrt{2}}\big]$ over $\mathbb{Q}$.

(20)

3. Show that $\mathbb{Q}\left[\sqrt[5]{3}\right]$ is a field.

(15)

4. Show that if f and g are monic polynomials over a ring $\mathbb{A}$ then so is $f(X)g(X)$.

What can we say about $f(X)-g(X)$?

(15)

5. Carefully, and in complete detail, define each of the following terms or expressions.

(a) Constructible number.

(b) Simple extension.

(c) Minimal polynomial

(d) Degree of an extension.

(e) α is algebraic over $\mathbb{Q}[\sqrt{2}]$.

(20)

6. Describe in detail how you can find a line segment of length $\sqrt{\alpha}$ given line segments of lengths 1 and α.

Remember that you should prove that your construction works.

(10)

7. (a) Write down a spanning set for $\mathbb{Q}[\sqrt{5}, \sqrt{17}]$ over $\mathbb{Q}$.

(b) Show that $\sqrt{17} \notin \mathbb{Q}[\sqrt{5}]$.

(c) Hence, or otherwise, show that your spanning set is a basis.

(25)

8. Give an outline of how we proved that it is impossible to trisect a general angle.

Your answer should include a description of the problem and a list of the major theorems used in the proof.

(15)

9. Find the multiplicative inverse of $1 - 2\sqrt[3]{2} - (\sqrt[3]{2})^2$ in the field $\mathbb{Q}[\sqrt[3]{2}]$.

(20)

10. (a) Show that given $f(X) \in \mathbb{K}[X]$ and $\alpha \in \mathbb{K}$: $f(X)$ is irreducible over $\mathbb{K}$ if and only if $f(X + \alpha)$ is irreducible over $\mathbb{K}$.

(b) What use is this?

(40)